



Nonlinear models to characterize cumulative gas production using the *in vitro* technique in *Brachiaria* grass silage harvested at three cutting ages

[Modelos não lineares para caracterizar a produção cumulativa de gás utilizando a técnica *in vitro* em silagem de capim *Brachiaria* colhida em três idades de corte]

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Abstract: The Brazilian ruminant production system, which relies on native or cultivated pastures, underscores the importance of silage for feed preservation in the tropical regions. This study aimed to evaluate different mathematical models for cumulative *in vitro* gas production in *Brachiaria decumbens* grass silages at three distinct cutting ages (56, 84, and 112 d). The *in vitro* gas production technique provided information on fermentation kinetics, were analyzed using mathematical models. The methodology included statistical tests to evaluate normality, independence, and heteroscedasticity of the residuals, as well as the application of various criteria for model selection. The results indicated that models such as Logistic, von Bertalanffy, Gompertz, Richards, Brody, and France were appropriate, but the France and Brody models provided the best overall fit. Based on the adopted criteria, the Brody model was found to be the most suitable for describing cumulative gas production across all cutting ages of *Brachiaria* grass. Cumulative gas production was highest at 56 days of growth, with the Brody model indicating superior production capacity during this period (191.14 mL/g dry matter). In conclusion, an appropriate choice of mathematical model is important for the accurate representation of gas production kinetics in *Brachiaria* grass silage.

Keywords: fermentation; tropical forage; model selection.

Resumo: O sistema de produção de ruminantes no Brasil, que se baseia em pastagens nativas ou cultivadas, ressalta a importância da silagem para a conservação do alimento nas regiões tropicais. Este estudo teve como objetivo avaliar diferentes modelos matemáticos para a produção cumulativa de gás *in vitro* em silagens de capim *Brachiaria decumbens* em três idades de corte distintas (56, 84 e 112 dias). A técnica de produção de gás *in vitro* forneceu informações sobre a cinética da fermentação, que foram analisadas utilizando modelos matemáticos. A metodologia incluiu testes estatísticos para avaliar a normalidade, independência e heterocedasticidade dos resíduos, bem como, a aplicação de diversos critérios para a seleção do modelo. Os resultados indicaram que

modelos como o Logístico, von Bertalanffy, Gompertz, Richards, Brody e France foram adequados, mas os modelos de France e Brody apresentaram o melhor ajuste geral. Com base nos critérios adotados, o modelo de Brody foi considerado o mais adequado para descrever a produção cumulativa de gás em todas as idades de corte do capim *Brachiaria*. A produção cumulativa de gás foi maior aos 56 dias de crescimento, com o modelo de Brody indicando capacidade de produção superior durante esse período (191,14 mL/g de matéria seca). Em conclusão, a escolha adequada do modelo matemático é importante para a representação precisa da cinética de produção de gás na silagem de gramíneas do tipo *Brachiaria*.

Palavras-chave: fermentação; forragem tropical; seleção de modelos.

1. Introduction

The Brazilian ruminant production system is predominantly based on the use of native or cultivated pastures, which often experience frequent water stress, leading to seasonal fluctuations in forage availability ⁽¹⁾. During periods of forage scarcity, it is essential to rely on alternative feed sources, with silage being a crucial tool for preserving unconventional grasses including species of the genus *Brachiaria* ⁽²⁾.

The *in vitro* gas production technique has emerged as a valuable tool for analyzing the fermentation kinetics of soluble, structural, and nonstructural feed components ⁽²⁾. Thus, it is possible to measure the ruminal digestion rate coupled with gas production and feed fermentation using *in vitro* gas production ⁽³⁾. However, these complex data must be interpreted using mathematical models that provide a simplified representation of the phenomenon under study ⁽⁴⁾.

Nonlinear mathematical models provide a more refined interpretation of the investigated phenomena by using various parameters with biological relevance ⁽⁵⁾. Thus, by fitting gas production to a nonlinear mathematical model, crucial information such as the gas production rate, colonization time, and asymptotic gas production can be obtained ⁽⁶⁾. Several authors ^(1, 2, 7, 8) have proposed various models that offer improved fit to gas production data, including the Gompertz ⁽⁹⁾ and Logistic and modified Logistic models ⁽¹⁰⁾, among others.

In this context, the present study aimed to evaluate the applicability of different mathematical models for fitting the cumulative gas-production kinetics of *in vitro* gases in *Brachiaria decumbens* grass silages at three distinct cutting ages.

2. Material and methods

2.1 Data

Data on cumulative gas production from *Brachiaria* grass silages at three cutting ages were used, as shown in Table 1.

Table 1. Cumulative gas production (mL/g) of dry matter after 6, 12, 24, 48, and 96 hours for *Brachiaria decumbens* grass silages harvested at 56, 84, and 112 days of growth.

Age of cutting (days)	Fermentation period (hours)				
	6	12	24	48	96
	Cumulative gas production (mL/g)				
56 days	6.91	28.93	94.17	147.03	182.12
84 days	6.58	18.34	69.50	125.92	167.80
112 days	8.43	27.36	78.20	127.92	165.17

2.2 Residual analysis

The Shapiro–Wilk test ⁽¹¹⁾ was used to assess normality, the Durbin-Watson test ⁽¹²⁾ was used to test independence, and the Breusch-Pagan test ⁽¹³⁾ was used to examine heteroscedasticity of the residuals.

2.3 Evaluated nonlinear models

The cumulative gas production of *B. decumbens* grass silages was fitted using the mathematical models presented in Table 2.

Table 2. Description of evaluated statistical models

Models	Parameter	Equations
<i>Brody</i>	3	$W_{(t)} = A[1 - be^{-kt}] + \varepsilon$
<i>Von Bertalanffy</i>	3	$W_{(t)} = A[1 - be^{-kt}]^3 + \varepsilon$
<i>Gompertz</i>	3	$W_{(t)} = Ae^{[1 - be^{-kt}]} + \varepsilon$
<i>France</i>	4	$W_{(t)} = A \left\{ 1 - e^{[-b(t-\lambda) - c(\sqrt{t} - \sqrt{\lambda})]} \right\} + \varepsilon$
<i>Logistic</i>	3	$W_{(t)} = \frac{A}{1 + e^{[-(b+kt)]}} + \varepsilon$
<i>Logistic Modified</i>	3	$W_{(t)} = \frac{A}{1 + e^{[2-4k(t-\lambda)]}} + \varepsilon$
<i>Bicompartmental Logistic</i>	5	$W_{(t)} = \frac{A_1}{1 + e^{[2-4k_1(t-\lambda)]}} + \frac{A_2}{1 + e^{[2-4k_2(t-\lambda)]}} + \varepsilon$
<i>Richards</i>	3	$W_{(t)} = A(1 - be^{-kt})^D + \varepsilon$
<i>Santos 2018</i>	3	$W_{(t)} = A \left(1 - be^{-\lambda e^{kt}} \right) + \varepsilon$
<i>Santos 2019</i>	4	$W_{(t)} = A_1(1 - b_1 e^{k_1 t})^3 + A_2(e^{b_2 e^{-k_2 t}}) + \varepsilon$
<i>Santos 2023</i>	4	$W_{(t)} = A_1 \{ (1 + e^{[2-4k_1(t-\lambda)])} \}^{-1} + A_2 \{ (1 + e^{[2-4k_2(t-\lambda)])} \}^{-1} + \varepsilon$
<i>Figueiredo 2023</i>	3	$W_{(t)} = A(1 - be^{-D}(1 - be^{-kt})^3) + \varepsilon$

Where: $W_{(t)}$ is the accumulated volume (mL) at time t ; A , total volume of gas produced (mL); A_1 , volume of gas produced by degradation of fraction $A + B_1$; A_2 , volume of gas produced by degradation of fraction B_2 ; k , specific gas production rate; k_1 , specific gas production rate by degradation of fraction $A + B_1$; k_2 , specific gas production rate by degradation of fraction B_2 ; D is the parameter shaping the curve over time (days); t , fermentation time; λ , latency phase; b and c , shape parameters, without biological interpretation; e , exponential; ε , experimental error associated with each observation.

2.4 Criteria for model selection

The adopted criteria to verify the quality of fit were:

The residual mean square (RMS) was calculated by dividing the sum of the squared residuals by the degrees of freedom of the residual $n - p$, that is:

$$RMS = \sum_{i=1} \frac{(y_i - \hat{y}_i)^2}{n - p}$$

This statistic represents the mean of the squared differences between the actual values (y_i) and predicted values in the model (\hat{y}_i), where n is the number of observations and p is the number of parameters used.

The mean absolute deviation (MAD) was obtained from the mean of the distances between each data point and the sample mean, defined as the mean of the absolute differences between the actual values (y_i) and predicted values in the model (\hat{y}_i) divided by n (number of observations), obtained using the following formula:

$$MAD = \frac{\sum |y_i - \hat{y}_i|}{n}$$

The information criterion (AIC) (14) and Bayesian information criterion (BIC) (15) are used to compare competing models and to increase the likelihood of selecting the model that best approximates the underlying data-generating process. In both cases, lower values indicate a better fit, and they are defined by:

$$AIC = -2\log L + 2(p + 1)$$

$$BIC = -2\log L + (p + 1)\log(n)$$

The mean absolute percentage error (MAPE) quantifies the average magnitude of prediction error expressed as a percentage of the actual values, calculated as the mean of the absolute percentage differences between the predicted and actual values using the following formula:

$$MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

Where, y_i is the actual value, \hat{y}_i is the predicted value, and n is the number of observations.

Penalizing adaptive likelihood (PAL) is a model-selection approach that introduces an adaptive penalty into the likelihood function, thereby discouraging overfitting and guiding the estimation of appropriate model complexity:

$$PAL = -2\ln L_n + n \ln(\hat{n}) \frac{\ln(r_n + 1)}{\ln(p_n + 1)}$$

Where r_n and p_n are the generalized likelihood ratios, n is the number of parameters, and \hat{n} is the largest number of parameters among the models considered.

A residual distribution plot was used to visualize the quality of the fit provided by each function and to assess whether the residuals exhibited patterns indicative of model misfit.

3. Results

Table 3 summarizes the parameter estimates for each model, accompanied by the evaluation criteria presented in Table 4, which together identify the model that best represents the average cumulative gas production curve observed during the 56-day experimental period.

All models exhibited normality of residuals ($p > 0.05$) according to the Shapiro-Wilk test. According to the Durbin–Watson test, all models had independent residuals. Regarding the Breusch-Pagan test, the *Von Bertalanffy* model ($p = 0.0289$) and *Gompertz* model ($p = 0.041$) showed evidence of heteroscedasticity; whereas the remaining models exhibited homoscedastic residuals ($p > 0.05$) for the 56-day cut.

Parameter estimates (Table 3) were all significant ($p < 0.05$) for the *Brody* and *Von Bertalanffy* models, indicating that these models adequately describe the average cumulative gas growth curve for the 56-day cut. In the *France*, *Logistic*, *Logistic Modified*, and *Santos* models,

only parameter A was significant ($p < 0.05$). For the *Gompertz* and *Richards* models, parameters A and K were significant ($p < 0.05$) and B was marginally significant ($p < 0.1$). However, in the *Figueiredo* model, none of the parameters reached statistical significance ($p > 0.1$).

Table 3. Parameter estimates for cumulative gas production (mL/g) over the 56-day period, where A represents the potentially degradable fraction, B the potentially degradable fraction under microbiota action without colonization time, and K the degradation rate constant.

Model	A	B	K
<i>Brody</i>	191.1483	1.2157	0.03454
<i>Von Bertalanffy</i>	179.7491	0.9129	0.0608
<i>Logistic</i>	168.0822	25.3333	0.1404
<i>Gompertz</i>	178.824	4.2386	0.0744
<i>Logistic Modified</i>	168.0823	0.0351	8.7772
<i>France</i>	183.6858	0.06011	-0.1841
<i>Richards</i>	181.3705	2.3980	0.0559
<i>Santos 2018</i>	164.5423	0.0328	0.1359
<i>Figueiredo</i>	233.9104	0.9348	0.0452

R^2 indicates the proportion of total variability in the response variable (cumulative gas volume) explained by the explanatory variable (time in hours) and was similar among the models (Table 4). R^2 values were consistently high (above 0.97), indicating strong agreement between the observed data and the model predictions. As shown in Figure 1, which includes the cumulative gas production curves from time up to 96 hours, had a mean of 91.83 mL/g and ranged from approximately 6.91 to 182.12 mL/g, corresponding to the fitted equations of the respective models for the 56-day cut.

However, the *France* model had the lowest Mean Absolute Deviation (MAD) value (Table 4), indicating the best-fit according to this criterion, followed by the *Figueiredo* and *Brody* models. Examining the sum of the Residual Mean Square (RMS) of the four models analyzed, the *Richards* model showed the best fit for growth estimates (Table 4). RMS serves as an indicator of fit quality because it is directly related to the variance of the residual errors; higher RMS values reflect greater error variability and therefore lower model adequacy.

For the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC), the *France* model showed the best fit with values of 37.66 and 35.70, respectively (Table 4). The *Brody* model performed best according to the PAL metric. The Mean Absolute Percentage Error (MAPE), was negative for all models evaluated for *Brachiaria* grass silage at the 56-day cut. Furthermore, the *France* model had the lowest MAPE baseline, indicating predictions closest to zero error.

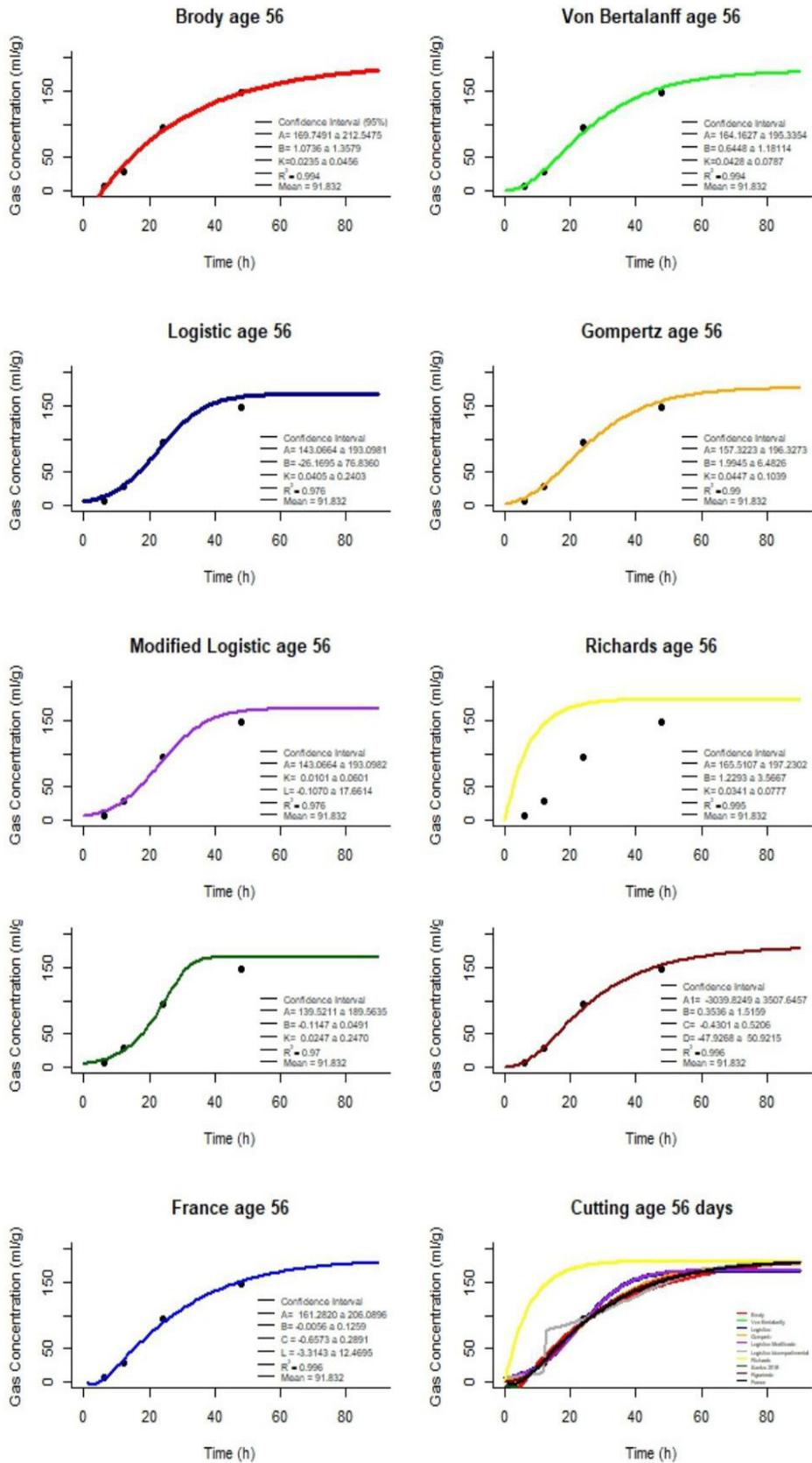


Figure 1. Cumulative gas production curves (mL/g) 56-day cutting age over time (hours).

Table 4. Selection criteria used to identify the most appropriate nonlinear model of cumulative gas production (mL/g) over the 56-day period.

Model	R ²	RMS	MAD	AIC	BIC	PAL	MAPE
<i>Brody</i>	0.994	58,659	3,7696	37.9663	36.4040	29,9662	-14.9831
<i>Von Bertalanffy</i>	0.994	59,690	4,4008	38.0538	36.4915	49,6285	-15.0269
<i>Logistic</i>	0.976	264,775	8,4679	45.5023	43.9401	37,3431	-18.7512
<i>Gompertz</i>	0.990	107,416	6,0349	40.9915	39.4292	Inf	-16.4957
<i>Logistic Modified</i>	0.976	264,775	8,4679	45.5023	43.9401	39,8395	-18.7512
<i>France</i>	0,996	73,9331	3,5025	37,658	35,7051	32,2559	-13,829
<i>Richards</i>	0,995	51,6443	4,1905	37,3298	35,7675	Inf	-14,6449
<i>Santos 2018</i>	0,970	325,9679	8,7880	46,5419	44,9796	41,6778	-19,2709
<i>Figueiredo</i>	0,996	85,1556	3,6713	38,3646	36,4117	Inf	-14,1823

Note: Residual Mean Square (RMS), Akaike information criterion (AIC), Bayesian information criterion (BIC), Mean Absolute Deviation (MAD), adaptively penalizing likelihood (PAL), and Mean Absolute Percentage Error (MAPE).

Table 5 presents the parameters for each model and the criteria used to evaluate the model that best described the mean cumulative gas growth curve for the 84-day cut. All models exhibited normality of residuals ($p > 0,05$), according to the Shapiro-Wilk test showed independent residuals based on the Durbin-Watson test. In relation to the Breusch-Pagan test, only *France* model ($p = 0,04867$) showed evidence of heteroscedasticity while other models exhibited homoscedastic residuals ($p > 0,05$).

Table 5. Parameter estimates for cumulative gas production (mL/g) over the 84-day period, where A represents the potentially degradable fraction, B the potentially degradable fraction degraded by microbiota without colonization time, and K the degradation rate constant.

Model	A	B	K
<i>Brody</i>	189.4062	1.1555	0.0247
<i>Von Bertalanffy</i>	170.6983	0.8814	0.0482
<i>Logistic</i>	164.7697	15.5338	0.0872
<i>Gompertz</i>	167.7781	4.0486	0.0586
<i>Logistic Modified</i>	164.769	0.0218	8.5163
<i>France</i>	174.4825	0.0501	-0.1809
<i>Richards</i>	172.4801	2.2932	0.0442
<i>Santos 2018</i>	-3.2830	0.0113	0.02159
<i>Figueiredo</i>	256.5095	0.9038	0.0383

The R² values were similar among the models (Table 6), all showing high values (> 0.98); however, the Santos (2018) model exhibited a markedly lower R² (0.76). Overall, the models demonstrated adequate agreement with the observed data as shown in Figure 2, which represents the cumulative gas production curves up to 96 hours, with an average of 77.628 mL/g, and ranged from approximately 6.56 to 167.80 mL/g, corresponding to the fitted equations of the respective models for the 84-day cut.

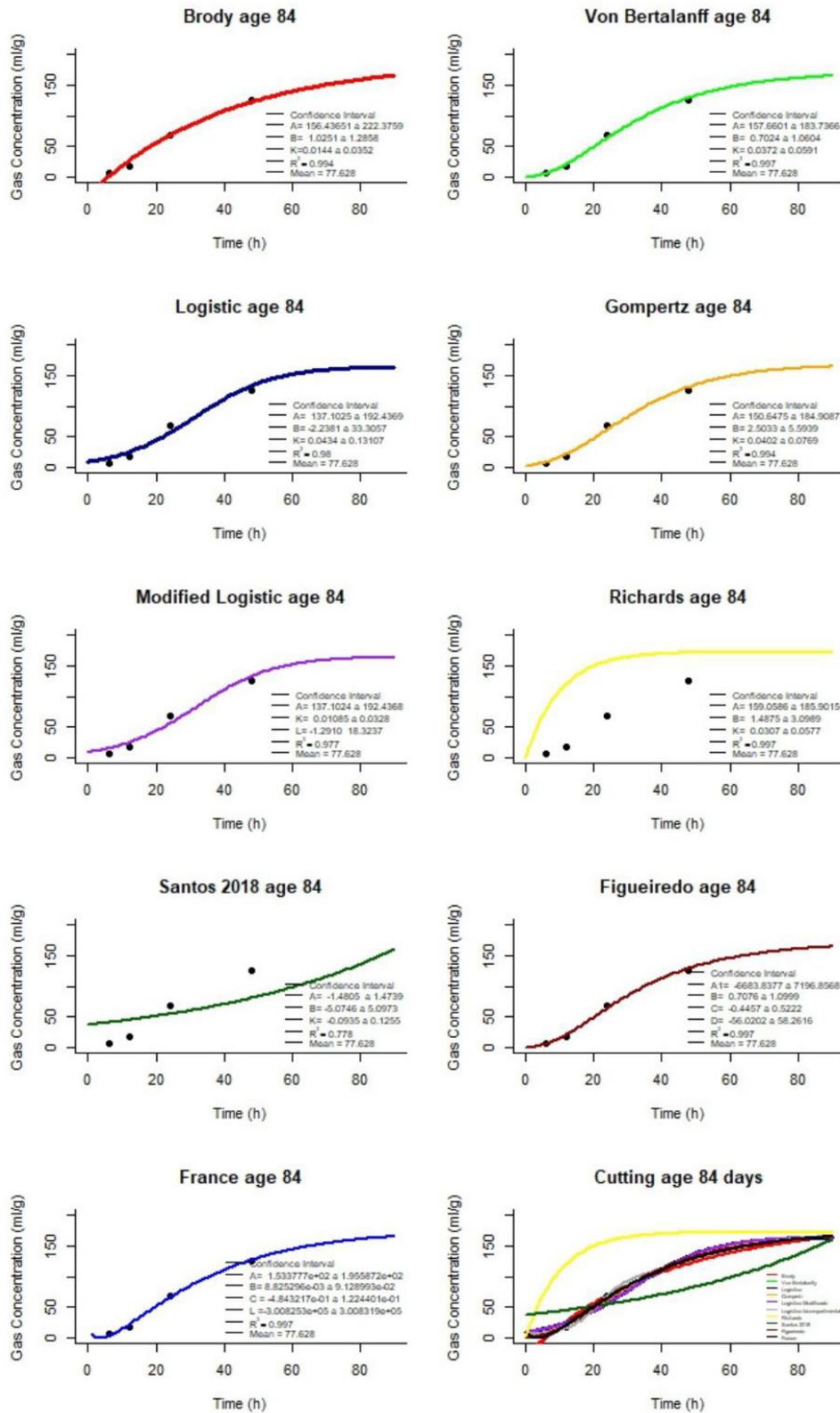


Figure 2. Cumulative gas production curves (mL/g) for the 84-day cutting age over time (hours).

However, the *France* model had the lowest MAD value (Table 6), indicating the best fit according to this criterion, followed by the *Richards* and *Figueiredo* models. Examining the sum of the Residual Mean Square (RMS) of the four models analyzed, the Richards model showed the best fit for gas-accumulation estimates (Table 6).

For the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC), France model again demonstrated the best fit, with values of 32.91 and 30.95, respectively (Table 6). In terms of PAL, France, Richards, and Brody models performed best. The MAPE was negative for all models evaluated for *Brachiaria* grass silage at the 84-day cut. Furthermore, the France model had the lowest MAPE baseline, indicating predictions closest to zero error.

Table 6. Selection criteria used to identify the most appropriate nonlinear model of cumulative gas production (mL/g) over the 84-day period.

Model	R2	RMS	MAD	AIC	BIC	PAL	MAPE
<i>Brody</i>	0.994	725,71	3,924	38.0033	36.4410	30,0032	-15.0016
<i>Von Bertalanffy</i>	0.997	28,437	2,8537	34.3465	32.7842	38,7225	-13.1732
<i>Logistic</i>	0.980	189,566	8,1493	43.8316	42.2694	38,9767	-17.9158
<i>Gompertz</i>	0.994	59,576	4,557	38.0442	36.4820	Inf	-15.0221
<i>Logistic Modified</i>	0.997	189,566	8,1494	43.8316	42.2694	38,5241	-17.9158
<i>France</i>	0,997	38,2998	2,4584	34,3694	32,4166	30,5982	-12,1847
<i>Richards</i>	0,997	22,9209	2,5605	37,3298	35,7675	Inf	-12,6340
<i>Santos 2018</i>	0,778	2117,372	26,0355	46,5419	44,9796	51,5136	-23,9487
<i>Figueiredo</i>	0,997	45,8686	2,6156	35,2711	33,3183	Inf	-12,6355

Table 7 presents the parameter estimates for each model and the criteria used to evaluate the model that best represents the data. All models exhibited normality of residuals ($p > 0,05$), according to the *Shapiro-Wilk* test and showed independent residuals based on the *Durbin-Watson* test. In relation to the *Breusch-Pagan* test, exhibited homoscedastic residuals ($p > 0,05$) for the 112-day cutoff.

Table 7. Parameter estimates for cumulative gas production (mL/g) over the 112-day period, where A represents the potentially degradable fraction, B the potentially degradable fraction degraded by microbiota without colonization time and K the degradation rate constant

Model	A	B	K
<i>Brody</i>	178.1226	1.1566	0.0292
<i>Von Bertalanffy</i>	165.9811	0.8117	0.0508
<i>Logistic</i>	159.7907	12.4783	0.0909
<i>Gompertz</i>	163.4376	3.6078	0.0611
<i>Logistic Modified</i>	159.7904	0.0227	5.7631
<i>France</i>	170.8596	0.04586	-0.1198
<i>Richards</i>	168.5694	1.9480	0.0442
<i>Santos 2018</i>	-1.0930	0.0038	0.0148
<i>Figueiredo</i>	236.5209	0.8503	0.0396

The R^2 values were similar among the models (Table 8), all showing high values (> 0.98); however, the Santos (2018) model exhibited a substantially lower R^2 (0.76). Overall, the models demonstrated adequate agreement with the observed data, as shown in Figure 3, which includes the cumulative gas production curves up to 96 hours. These curves had a mean of 81.416 mL/g and ranged from approximately 8.43 to 165.17 mL/g, corresponding to the fitted equations of the respective models for the 112-day cut.

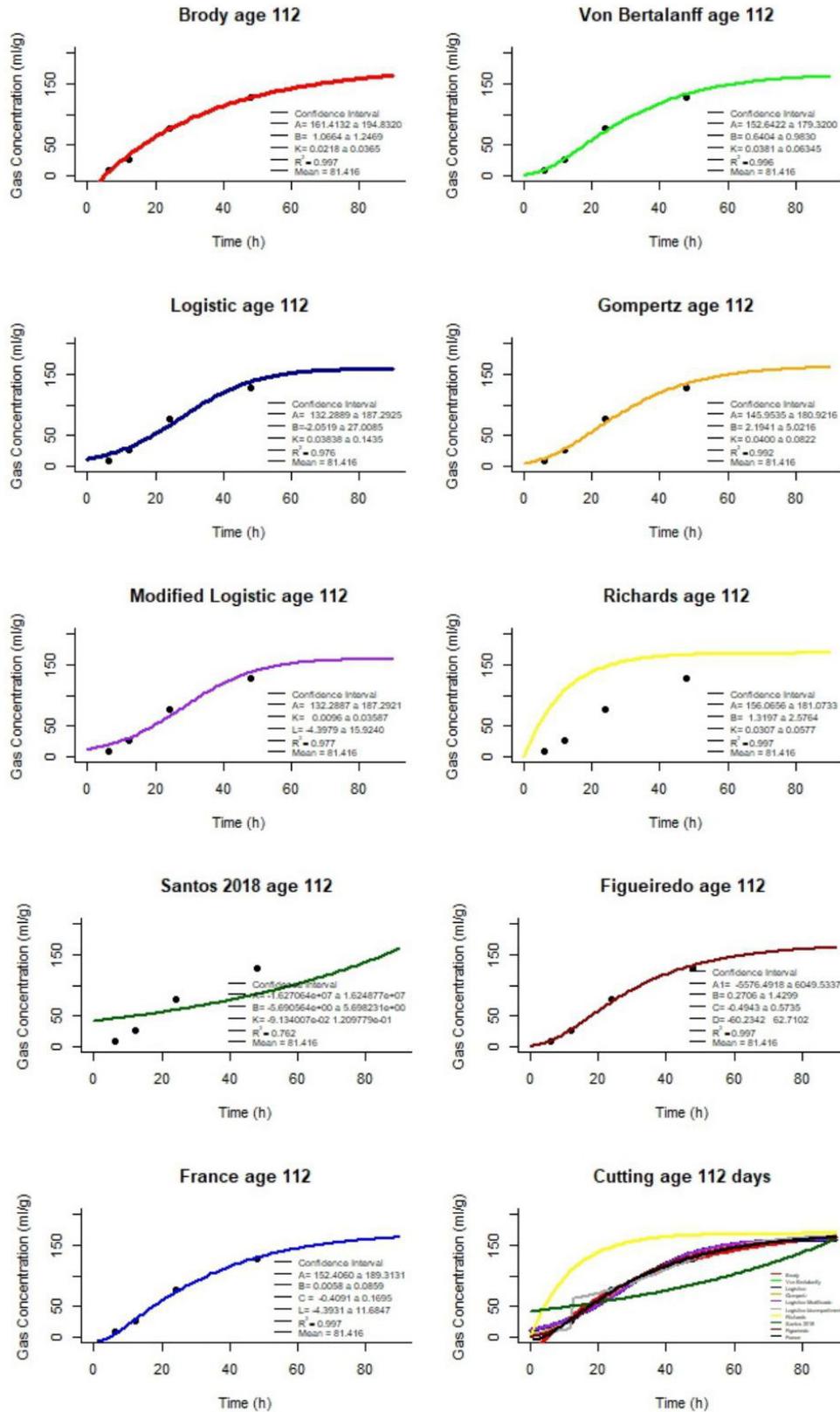


Figure 3. Cumulative gas production curves (mL/g) for the 112-day cutting age over time (hours).

However, the *France* model had the lowest MAD value (Table 8), indicating the best fit according to this criterion, followed by the *Brody* and *Figueiredo* models. Examining the sum of the Residual Mean Square (RMS) of the four models analyzed, the France and Brody models showed the best fit for gas-accumulation estimates (Table 8). For the Akaike information criterion (AIC) and Bayesian information criterion (BIC), France model again demonstrated the best fit, with values of

32.91 and 30.95, respectively (Table 8). In terms of PAL, the models *France*, *Richards*, and *Brody* performed the best. The MAPE was negative for all models evaluated for *Brachiaria* grass silage at the 112-day cut.

Table 8. Selection criteria used to identify the most appropriate nonlinear model of cumulative gas production (mL/g) over the 112-day period.

Model	R ²	RMS	MAD	AIC	BIC	PAL	MAPE
<i>Brody</i>	0,997	23,979	2,4335	33.4938	31.9315	25,4938	-12.7469
<i>Von Bertalanffy</i>	0,996	34,092	3,3302	35.2532	33.6910	45,7212	-13.6266
<i>Logistic</i>	0,976	202,147	8,3824	44.1529	42.5906	Inf	-18.0764
<i>Gompertz</i>	0,992	68,779	4,7947	38.7624	37.2002	Inf	-15.3812
<i>Logistic Modified</i>	0,977	271,068	8,3824	44.1529	42.5906	38,9093	-18.0764
<i>France</i>	0,998	23,9788	2,1331	32,9077	30,9549	29,0381	-11,4538
<i>Richards</i>	0,997	103,3533	5,6742	32,9774	31,4151	Inf	-12,4887
<i>Santos 2018</i>	0,762	2070,879	26,1675	55,7865	74,2243	51,4916	-23,8932
<i>Figueiredo</i>	0,997	51,4425	2,8876	35,8445	33,8917	Inf	-12,9222

Given the high determination coefficients obtained and the 5% significance level of the parameters, the *Brody* model (Figure 4) can be considered well-suited to all cuts of *Brachiaria* grass, as supported by the model equation and the corresponding R² values presented in Table 9.

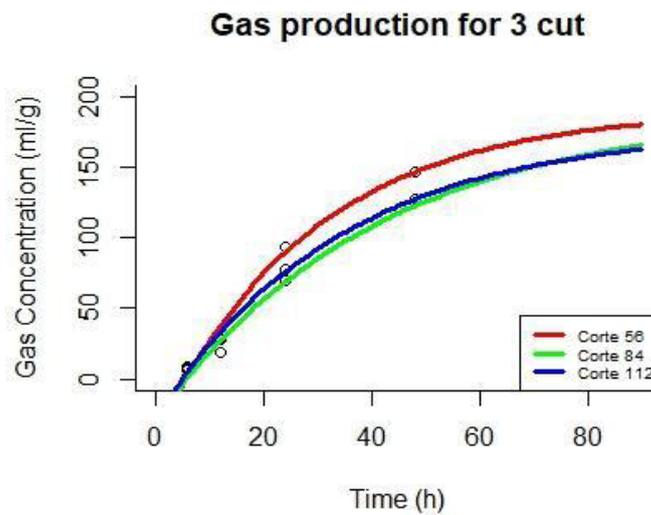


Figure 4. Cumulative gas production curves (mL/g) for the 56-, 84-, and 112-day cutting ages over time (hours), based on observed data and fitted using the Brody model

Table 9. Equations generated by regression analysis for the *Brody* model applied to cumulative gas production data from *Brachiaria decumbens* grass silages harvested at 56, 84, and 112 days of growth.

Age	Equation	R ²
Cut 56	$W(t) = 191.14(1 - 1.2157e^{-0.034454t})$	0.99
Cut 84	$W(t) = 189.40(1 - 1.1554e^{-0.02477t})$	0.99
Cut 112	$W(t) = 178.12(1 - 1.1566e^{-0.02919t})$	0.99

4. Discussion

Models applied to ruminal digestion kinetics provide a straightforward interpretation of the phenomena under study using parameters with biological meaning ⁽⁶⁾. Numerous models have been proposed and tested across different substrates ^(4, 16, 17), each based on distinct assumptions

and mathematical approaches, and each describing changes in the system as a function of incubation time⁽³⁾. Nevertheless, models with varying mathematical structures may yield different results for the same gas- production curve, as previously observed.

At the 56-day harvest, the differences among the model parameters were less pronounced than those observed at 84 and 112 days. However, the indices clearly show that Santos (2018) model produced results that were distinct from the other models. Although the Richards model appeared visually different in the plotted curves, the indices did not reveal this distinction as clearly they did for the Santos (2018) model.

Based on the R^2 criterion, the models showed comparable performance after 56 d. However, at 84 and 112 d, the Santos (2018) model performed markedly worse than the others, indicating that it is not a suitable option under this criterion. A similar situation occurred with the RMS, where the Santos (2018) model consistently produced discrepant values across all harvests whereas the Logistic and Modified Logistic models also showed higher RMS values relative to the remaining models.

Across all the evaluated criteria, Santos (2018) model consistently yielded inferior performance, demonstrating that it is not suitable for this dataset. This becomes even more evident when the combined function plots are considered. Although the R^2 values of the Logistic and Modified Logistic models were comparable, they did not show equivalent performance across the remaining criteria.

Multiple indices were applied to ensure a more rigorous evaluation as it was necessary to verify whether they provided consistent information. According to Mello (4), a negative MAPE indicates an underestimation of the predicted parameters. All models presented negative values, suggesting that the parameter values were underestimated in every case. The main difference lies in the absolute magnitude of these values, with smaller values reflecting a better fit.

It is important to note that the Brody model produced the best overall fit. Despite its simple mathematical formulation, it was able to describe the gas-production dynamics more effectively. Based on the Brody model, the material harvested at 56 days exhibited the highest gas production capacity (191.14 mL/g of dry matter), followed by silages from grass harvested at 84 and 112 days, with 189.12 mL/g and 178.12 mL/g of dry matter, respectively. Castro⁽¹⁸⁾ reported that the effective degradation of dry matter decreased with advancing plant age in Tanzanian grass silages produced at 42 and 126 days. Similarly, Silva⁽¹⁹⁾ observed that gas production declined with increasing maturity of elephant grass harvested at 60 and 90 d.

The results indicate that the effective degradability of dry matter declined with advancing age in *Brachiaria* grass⁽²⁾ and elephant grass silages⁽²⁰⁾. This trend may be explained by the strong correlation between gas production and dry matter degradability as demonstrated by Ribeiro et al.⁽²¹⁾ in *Andropogon* grass silages of different ages. Therefore, as the cutting age increased, the silage decomposition capacity decreased owing to an increase in the stem-to-leaf ratio, which in turn increased the proportions of cellulose, hemicellulose, and lignin, thereby reducing the fraction of potentially digestible nutrients such as soluble carbohydrates and proteins, ultimately leading to a marked decline in digestibility. Additionally, material losses during silo fermentation further reduce gas production compared to fresh forage. The greatest gas production capacity *in vitro* was observed at a cutting age of 56 days, reflecting a higher proportion of soluble carbohydrates in the cellular content and, consequently, greater substrate availability for ruminal microorganism.

Therefore, cutting the *B. decumbens* grass at this stage is recommended for optimal ensiling performance.

5. Conclusion

Considering all selection criteria based on the parameters, the Brody model showed the best overall fit to the cumulative gas production curves of *B. decumbens* silages harvested at 56, 84, and 112 days of growth. The Santos 2018, Logistic, and Modified Logistic models did not adequately represent the data and are therefore not recommended for this type of silage.

Conflict of interest statement

The authors declare no conflict of interest.

Data availability statement

The complete dataset supporting the results of this study was published in the article itself and in the repository: <https://hdl.handle.net/1843/BUOS-98LFGY>.

Author contributions

Conceptualization: Moreira, G. R. Validation: Moreira, G. R. Methodology: Nascimento, E. R. Data curation: Magalhães, F. A. and Gonçalves, L. C. Formal analysis: Nascimento, E. R., Figueira, M. V. and Costa, A. C. F. Writing (original draft): Nascimento, E. R., Figueira, M. V., Silva, D. M. and Castigo, M. J. Writing (review and editing): Moreira, G. R., Cunha Filho, M. and Oliveira, C. E. N. Writing (review and editing – English version): Figueira, M. V. and Silva, D. M.

Generative AI use statement

No Generative Artificial Intelligence tools were used in the preparation of this manuscript.

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