Diatonic Scale Interaction and Intervallic Content in Contrapuntal Polydiatonicism: On the Applicability of Set Theory

Interação entre Escalas Diatônicas e Conteúdo Intervalar no Polidiatonismo Contrapontístico: Sobre a Aplicabilidade da Teoria dos Conjuntos

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Abstract: Interval classes (ICs) can be generated not only through the combination of pitches within a single pitch-class set, but also through the interaction between pitches from two distinct sets (scales), thereby revealing the potential for generating various intervals across different sets. In actual music, such inter-set interaction is primarily manifested in the construction of harmonic intervals, which can, particularly in polyphonic polytonal music, arise from melodic lines of different scalar origins, and the comparison between two or more instances of such interactions. One particularly representative structure is contrapuntal polydiatonicism, which is developed based on the diatonic scale. This article develops a theoretical framework for the Directed Pitch Interval Class (DPIC) vector based on concepts from set theory, incorporating the notion of ordered interval classes. The framework is applied to the analysis of contrapuntal polydiatonicism, involving twelve different tonal relationships, and demonstrates that their intervallic content distributions differ significantly. Certain interval classes appear in greater numbers within specific combinations; this distributional feature may influence a composer's choice of tonal relationships within polydiatonic structures.

Keywords: diatonic scale, contrapuntal polydiatonicism, polytonality, set theory, intervallic content

Resumo: Classes de intervalos (CIs) podem ser geradas não apenas por meio da combinação de notas dentro de um único conjunto de classes de notas, mas também pela interação entre notas provenientes de dois conjuntos distintos (escalas), revelando, assim, o potencial para a geração de diversos intervalos entre conjuntos diferentes. Na música real, tal interação entre conjuntos manifesta-se principalmente na construção de intervalos harmônicos, os quais podem, particularmente na música polifônica politonal, surgir de linhas melódicas de origens escalares distintas e da comparação entre duas ou mais instâncias dessas interações. Uma estrutura particularmente representativa desse fenômeno é o polidiatonismo contrapontístico, desenvolvido com base na escala diatônica. Este artigo propõe uma estrutura teórica para o vetor de Classe de Intervalo de Nota Direcionado (DPIC, na sigla em inglês), fundamentada em conceitos da teoria dos conjuntos e incorporando a noção de classes de intervalos ordenados. A estrutura é aplicada à análise do polidiatonismo contrapontístico envolvendo doze diferentes relações tonais, demonstrando que suas distribuições de conteúdo intervalar diferem significativamente. Certas classes de intervalos aparecem com maior frequência em combinações específicas; essa característica distributiva pode influenciar a escolha do compositor quanto às combinações tonais dentro de estruturas polidiatônicas.

Palavras-chave: escala diatônica; polidiatonismo contrapontístico; politonalidade; teoria dos conjuntos; conteúdo intervalar.

Submitted on: June 15, 2025
Accepted on: July 29, 2025
Published on: September 2025

Introduction

Throughout twentieth-century music theory literature, polytonality has remained a concept that is difficult to define and highly contested, provoking extensive debate concerning its analytical, historical, and perceptual implications (Chang; Salleh; Sun, 2024; Médicis, 2005). It is commonly understood as a form of polyphonic texture in multi-voiced music in which two or more tonalities or keys coexist simultaneously (Gut, 1976; Latham, 2011; Randel, 2003; Whittall, 2001). While it has been regarded as a significant musical language for composers such as Darius Milhaud, Alfredo Casella, Béla Bartók, Igor Stravinsky, Karol Szymanowski, and Benjamin Britten, it has also been interpreted as a continuation or dissolution of tonality, a manifestation of conservatism or avant-gardism, or even as a purely imaginary construct or a perceptible musical phenomenon (Delaere, 2012). In practice, the definition of polytonality exhibits considerable flexibility, extending beyond the strict simultaneity of multiple keys to encompass more subtle tonal interweaving across independent parts or musical layers.

Darius Milhaud (1923) remarked that the formation of the principle of polytonality did not represent a break from tradition but was deeply rooted in the practice of traditional counterpoint: "The day when canons were admitted at intervals other than the octave marked the birth of the principle of polytonality." Danièle Pistone (2011) extended this view, further emphasising that the earliest models of polytonality emerged directly from contrapuntal practice, and that there exists a profound continuity between the two, both in terms of technical development and conceptual foundation.

Today, it is necessary to seriously revisit the nature of this long-standing tradition for the insights it may offer into the study of polytonality: What, after all, is counterpoint? According to Markand Thakar (1990), "the counterpoint is a unit resulting from the *conjunction* of two primordial lines," and "if counterpoint is lines joining and the study of counterpoint begins with the study of the essence of line, then it continues with the study of the essence of *conjunction*" (p. 43). These statements reflect his view on the essential nature of contrapuntal structure. In other words, while counterpoint investigates the logic of melodic lines, it has never ignored the vertical harmonic structures generated as *conjunct lines*. As Wallace Berry (1987) points out, the meaning of counterpoint or contrapuntal lies in its designation of a condition of interaction among multiple voices—an interaction that involves intervallic content, directional motion, rhythm, and other diverse qualities or parameters: "Counterpoint or contrapuntal denote a condition of interlinear interaction involving intervallic content, direction, rhythm, and other qualities or parameters of diversification" (p. 197). This once again emphasises that counterpoint is not merely a method of melodic organisation, but a practice of constructing complex relationships among multiple voices.

However, one fact that cannot be overlooked is that nearly all major scholars of polytonality—such as Charles Koechlin (1925), Alfredo Casella (1924), Jens Rosteck (1994), Peter Kaminsky (2004), and Daniel Harrison (1997)—have emphasized its value in terms of linear melody and horizontal structure, yet have generally failed to provide a sufficiently systematic account of the profound transformations it introduces into vertical harmonic structures. In this regard, the remarks by Percy Scholes and John Ward on polytonalists remain illuminating:

> The polytonalists appear to claim that the value of their work lies in the significance of the horizontal lines as such, and that the vertical element is quite disregible. No such claim has been previously made in the history of music, even by the most advanced ultra-contrapuntalists. (Scholes; Ward, 1970, p. 449)

As Danièle Pistone (2005) has pointed out, polytonality was born in an era obsessed with harmony, yet it exhibited a contrary tendency. Within the continually evolving theoretical framework, contrapuntal polytonality came to be seen as a mere piling-up of tonalities—often difficult to control—and some even concluded that, even after a thorough study of specific works, it remains hard to formulate practical compositional principles for polytonality (Rosteck, 1994). Although some scholars, such as Franck Jedrzejewski (2011), J. Philip Lambert (1991), Marc Rigaudière (2011), and José Martins (2020), have extended Milhaud's observations on chordal structures in polytonality to the study of complex sonorities within scales and harmonies, such research has rarely undertaken a systematic examination of contrapuntal polytonality. In fact, the constructive relationships between tonal layers inevitably give rise to interaction; vertical and horizontal dimensions, as two interrelated planes coexisting in the same musical space, cannot be understood in isolation. In the process of analysis, polytonal compositions already imply the collisions between independent harmonic streams arising from tonal conflict (Tymoczko, 2011, p. 374).

So, is the choice of tonalities in the compositional process of contrapuntal polytonal music truly arbitrary? Departing from previous studies that often remain on the descriptive level of individual work analyses, this article introduces concepts from set theory concerning pitch intervals and interval classes, focusing specifically on a type of contrapuntal polytonality based on the diatonic scale—*contrapuntal polydiatonicism*. More specifically, it analyzes the distribution of harmonic intervals generated through the contrapuntal intervallic pairings of diatonic scales under different tonic relationships, to reveal the decisive role that tonal selection plays in shaping vertical contrapuntal structures. This, in turn, aims to provide more systematic theoretical support for understanding the structural mechanisms behind the composition of diatonic-based polytonality.

Literature Review

What is Contrapuntal Polydiatonicism?

In the early twentieth century, as compositional styles became increasingly diverse, early writings on polytonality were primarily produced by scholars such as Charles Koechlin, Alfredo Casella, and

Darius Milhaud. Approaching the subject from multiple angles—formal construction, tonal relationships, textural organisation, free counterpoint, and harmonic expansion—they sought to capture the richness of the phenomena produced by superimposing chordal and scalar materials and to explore how polytonality is formed and operates. Based on the textural characteristics displayed in polytonal music, Koechlin (1928, pp. 258–266) divided it into two basic types: *harmonic polytonality* and *contrapuntal polytonality*.[1] Koechlin noted that harmonic polytonality is characterised by the vertical superposition of several tonal harmonies, typically forming stratified sonorities through chordal groups or sound blocks, whereas contrapuntal polytonality is manifested when several voices, each proceeding in its tonality, intertwine and overlap to create a polyphonic effect.

Accordingly, the harmonic polytonality and contrapuntal polytonality present different analytical profiles. The former can be more readily accommodated within traditional harmonic analysis, although it often clashes with highly extended chords or modal constructions.[2] The latter, though its melodic lines are clear, is usually deemed difficult to analyse effectively with conventional harmonic tools because of its looser structure and intricate melodic interweaving.

Specifically, Alfredo Casella, in his discussion, also acknowledged the value of harmonic polytonality in terms of sonic expressiveness and theoretical analysis. However, he remained cautious toward melodic polytonality, which bears the same meaning as contrapuntal polytonality (Casella, 1924). He argued that, apart from certain compositional practices by Milhaud, there have thus far been few convincing results achieved through this technique, making it difficult to grant it the same status as harmonic polytonality. Even more telling is the fact that, when

1 The term harmonic polytonality was first coined by Émile Vuillermoz in his review of Casella's *Notti di maggio*, see (Vuillermoz, 1914); Alfredo Casella, by contrast, referred to contrapuntal polytonality as melodic polytonality, see (Casella, 1924).

2 The most prominent case being the debate over *Petrushka*'s chordal structure—see Richard Taruskin (1996), Pieter C. Van Den Toorn (1983), and Arthur Berger (1963)—and Ann McNamee's rebuttal of Szymanowski's polytonal language using the Podhalean scale, see (McNamee, 1985).

discussing his works involving contrapuntal polytonality, Milhaud himself admitted that the harmonies produced often defy analysis, characterising the resulting outcome as atonal in nature:

> It is noteworthy that in most instances of these polytonal counterpoints made from diatonic melodies, the harmonies arising from vertical conjunctions of tones are unanalysable, and the result is hence atonal. (Milhaud, 1923)

Since Casella (1924) first introduced the concept of *polydiatonicism*, the term has seldom been adopted in academic discourse to describe polytonal structures based on diatonic scales. Theorists have tended to favour alternatives such as *contrapuntal polytonality* or *polymodality*. Still, this substitution not only fails to clarify the terminology of polytonality, but it also obscures the specific semantic features that polydiatonicism is meant to denote. Philippe Malhaire (2011) redefined the term with greater precision, characterising it as a compositional approach in which the resulting harmonic effect appears atonal. However, the melodic lines remain grounded in diatonic scales. He further argued that, compared to contrapuntal polytonality, the term polydiatonicism offers a clearer expression, as it avoids misleading analysts into assuming that the work adheres to vertical harmonic rules within the traditional tonal system (Malhaire, 2013, p. 35).

Malhaire accurately grasped the core feature of polydiatonicism—namely, that a polytonal structure must be based on diatonic scales—but he did not fully clarify the essential distinction between this concept and contrapuntal polytonality. According to the binary framework proposed by Koechlin, which differentiates between harmonic polytonality and contrapuntal polytonality, the latter emphasises the independent interweaving of melodic lines and their contrapuntal pitch relationships. In contrast, the former focuses on vertical chordal structures. The distinction lies in the respective reference systems: texture for the one, pitch material for the other. Casella, as the originator of the term *polydiatonicism*, never conflated it with *melodic* *polytonality* or *contrapuntal polytonality*, as these terms concern different analytical dimensions. Serge Gut (1976, p. 821) classified Milhaud's music as contrapuntal polytonality based on its textural features, while Malhaire focused on the scalar properties of the melodic material. Although both perspectives reveal essential characteristics of Milhaud's music, their points of emphasis differ. Simply put, harmonic polytonality may also be entirely based on diatonic scales, and contrapuntal polytonality does not necessarily require the use of diatonic materials. Richard Taruskin (2010, p. 581) confirms this point: "both melodically and harmonically, Milhaud's style can be seen as a paradigm of *polydiatonicism*."

The meaning of counterpoint or contrapuntal lies in its indication of an interactive state among multiple voices—a state that involves intervallic content, directional motion, rhythm, and various other attributes or parameters (Berry, 1987, p. 192). Accordingly, this article adopts the term—*contrapuntal polydiatonicism*, emphasizing two main aspects: first, that the relationship between voices is fundamentally contrapuntal, focusing on the independence and interweaving of melodic lines rather than the vertical combination of traditional chordal units; second, that the scalar construction is grounded in the pure diatonic major-minor system, free from regional or artificial modes, thereby maintaining the purity of the scalar material.

Theories and Analyses of Polytonal Music

With the ongoing development of post-tonal theory, research on tonal expansion has significantly contributed to the theoretical deepening and accumulation of knowledge in this field. For instance, Gordon Cyr (1971), in his scholarship of Charles Ives's *Fourth Symphony*, explored the hierarchical relationships within polytonal structures and proposed that *intervallic similarity* between melodies contributes to overall unity. Kenneth Hicken (1974), using Schenkerian analysis to examine the theme of Schoenberg's *Variations for Orchestra*, introduced the concept of *fused-bitonal* *progression* to describe the layered structures resulting from tonal superposition. Constant Vauclain (1981), in his study of Bartók's works, proposed the notion of *polymodal chromaticism*, emphasising the binary oppositions formed through scalar counterpoint and the specific pitch distances between tonal centres. By discarding traditional diatonic scale-degree functions, José Martins (2015) proposed the *Dasian space* model, which is based on chromatic space. Building on this model and inspired by Alfredo Casella's notion of *simultaneous modulation*, Martins (2017, 2020) further developed an analytical model of harmonic distance known as *scalar dissonance*. This framework offers a means of quantifying the harmonic tension and interaction between polymodal scales, thereby overcoming the limitations of traditional tonal centres and conventional methods of measuring harmonic distance.

Although these studies differ in focus, their methodologies remain grounded in specific works or isolated concepts, lacking a systematic investigation into the vertical distribution of intervals within polytonal scales. As a result, they fail to reveal the internal hierarchical relationships and deeper mechanisms of interaction from the perspective of structural generation. This theoretical limitation is also evident in compositional analysis: "composers may prefer to select scales with the most distant tonal relationships to emphasize distinctions between different tonal regions and minimize shared tones; on the other hand, they sometimes choose scales with shared tones to create sonorities akin to synthetic or aggregate scales" (Benjamin, Horvit e Nelson, 2008, p. 210). This reflects a broader issue in current research on polytonal music: the selection of tonal relationships lacks a stable and systematic theoretical framework. Analytical approaches often remain at the level of empirical judgment and subjective speculation, without developing a structural perspective that can explain the generative mechanisms involved.

Since the mid-20th century, set theory has been widely applied in the analysis of atonal music, with particular emphasis on the intervallic content and structural relationships embedded within sets (Babbitt, 1962, 1965). Especially after Allen Forte (1973) systematically modelled the various interval classes and intervallic contents within sets through set classes and interval-class vectors, the theoretical framework of this approach gradually matured. Subsequently, John Rahn (1987) and Michiel Schuijer (2008) further expanded the applicability of set theory in explaining pitch relations and structural organisation, noting that set theory imposes no limit on the number and nature of ties between tone combinations. These studies not only reinforced the effectiveness of set theory as a tool for analysing atonal music but also demonstrated that the statistical structure of intervallic distribution holds significant theoretical potential for the analysis of tonal and polytonal music as well.

Polytonalists have also attempted to apply set theory in the analysis of polytonal music. Keith Daniel (1982) provides one of the few examples, but he ultimately pointed out that the direct application of set theory to explain polytonal structures has limited utility. He preferred to conduct localised set-theoretical analyses of individual works or passages, rather than developing a theoretical framework applicable to the overall structure of polytonal music. A more advanced analysis of Milhaud's *Second Chamber Symphony* (Op. 49) was provided by Daniel Harrison (1997). Responding to Daniel's concerns, Harrison employed the concept of sets to examine the structural challenges presented by polytonal composition and attempted to provide solutions. He emphasised that set theory can not only reveal the complex abstract relationships within polytonality but also provide novel analytical perspectives. However, he also cautioned analysts to be mindful of the fundamental distinction between "sleight-of-hand" bitonal treatments and structurally grounded polytonality based on the circle of fifths and set transformations.

By contrast, while set theory has reached a relatively mature and sophisticated status in the analysis of atonal music, its direct application to the study of polytonality still faces numerous challenges and limitations, especially when dealing with diatonic pitch-class sets. Rather than adopting established set-theoretical models wholesale, this study draws inspiration from their conceptual framework and repurposes certain tools to address the specific challenges posed by contrapuntal polydiatonism. Accordingly, I propose shifting the analytical focus toward understanding contrapuntal intervals within contrapuntal polydiatonicism. In this study, offering a preliminary exploration of the contrapuntal intervals and their distribution arises from the interaction between diatonic scales. This exploration will be guided by a set theory relevant theoretical considerations, as elaborated in the following section.

Methods: Adapting Set Theory to Polydiatonic Contexts

Polydiatonic Counterpoint Between Two Diatonic Scale Sets

Generally speaking, set theory primarily focuses on the structural coherence of pitch combinations in atonal music (Schuijer, 2008, p. 4). In previous scholarship, this abstract treatment of pitch itself has played a significant role in the analysis of atonal works. However, attempting to directly or entirely apply this theoretical system to the analysis of polydiatonic or polytonal structures presents numerous difficulties and may even lead to ambiguities. One of the most prominent issues is that, within set theory, diatonic scales from different keys (such as C major and G major) are categorised as the same pitch-class set type, 7-35, and share an identical *prime form*. The reason for this is quite apparent: set theory is grounded in the concept of the twelve pitches of the chromatic scale, rather than the concept of scale degrees within diatonic systems. As a result, it overlooks distinctions in actual pitch names and tonal functions, thereby diminishing the semantic and harmonic functions of scale degrees that tonal analysis relies upon.

Therefore, in this study, *the two distinct diatonic scales in a bitonal context are treated as two unordered pitch-class sets, without consideration of any prime form*, and are referred to as set A and set B, respectively. Given the importance of voice positioning in contrapuntal polydiatonicism, set A is consistently defined as the C major diatonic scale used in the lower voice, serving as the unified fixed-scale reference throughout the analysis; set B is selected from any of the twelve major scales (C, C♯, D, D♯, E, ..., B), and functions as the contrapuntal-scale, representing the key that appears in the upper voice.

For example, in a polydiatonic contrapuntal setting involving C major (C, D, E, F, G, A, B) and F major (F, G, A, B♭, C, D, E), the two diatonic scales can be represented as the following unordered sets:

$$A = \{0, 2, 4, 5, 7, 9, 11\} \qquad B = \{0, 2, 4, 5, 7, 9, 10\}$$

It is essential to clarify that the concept of unordered pitch-class sets here is not intended to blur the boundary between *polytonality* and *polymodality*. Instead, it follows a widely accepted understanding in polytonality studies.[3] Therefore, the contrapuntal relationship between set A and set B constitutes the complete range of possible pitch-class combinations within a given set of tonalities, representing a *basic form* of *contrapuntal polydiatonicism*. For example, when discussing other polydiatonic combinations whose tonic centres are a perfect fourth apart (e.g., G–C, B♭–E♭, etc.), their structure is entirely analogous to that of C–F. They may thus be regarded as transpositional variants of this *basic form*, sharing all structural features—except pitch-name labels—such as intervallic distribution and contrapuntal potential.

The Problem of Voice Displacement and Unordered Interval Classes

In traditional music theory, intervals are typically classified as melodic intervals (two tones sounding successively) and harmonic intervals (two tones sounding simultaneously). By contrast, the concept of pitch interval used in set theory is an abstract definition

3 For instance, scholars generally agree that most of Milhaud's polytonal music does not reflect tonal relationships in the traditional sense but rather modal ones; in fact, analyzing Milhaud's polytonality from a modal perspective has proven effective, see Jeremy Drake (1989, p. 201) and Deborah Mawer (1997, p. 18).

grounded in mathematics, referring to the number of semitones between two pitches (Straus, 2016, p. 20). Since this definition does not depend on the order of appearance or whether the tones sound simultaneously, it can theoretically encompass both melodic and harmonic intervals in the traditional sense.

In set theory, to form intervals, each pitch element in a given pc set can be paired with other pitch elements within the set. Based on octave equivalence, the theory establishes the classification concepts of *ordered interval class* (OIC) and *unordered interval class* (UIC) (Rahn, 1987, pp. 25–29). The so-called ordered interval class refers to an intervallic distance between two pitch classes within the twelve-tone equal temperament system that includes a directional component—represented on the clock face as the shortest distance in the clockwise direction, with values ranging from 0 to 11.[4] Each value corresponds to a specific interval category—for example, OIC0 represents unison and octave, OIC1 a semitone, OIC7 a perfect fifth, and so on—thus covering all twelve possible ordered intervals:

$$\text{for } i(a,\ b);\ f(b) = \begin{cases} b, & \text{if } b \geq a \\ b + 12, & \text{if } b < a \end{cases}$$

$$\text{OIC} = f(b) - a$$

Unordered interval classes are derived by inverting and reducing ordered interval classes, treating intervals that are inversions or complements of each other as equivalent and grouping them into the same category. The smallest intervallic distance is used to represent each class:

$$\text{UIC }(a,\ b) = \min\left(\text{OIC}(a,\ b), \text{OIC}(b,\ a)\right)$$

4 Milton Babbitt usually referred to similar concepts as directed pitch interval class or directed distance; see (Babbitt, 1965, 1962).

Thus, unordered interval classes include only six basic categories (IC1–UIC6): IC1 represents the minor second and major seventh (m2/M7); IC2 represents the major second and minor seventh (M2/m7); IC3 represents the minor third and major sixth (m3/M6); IC4 represents the major third and minor sixth (M3/m6); IC5 represents the perfect fourth and perfect fifth (P4/P5); and IC6 represents the tritone, which remains invariant under inversion. This classification system excludes information about unisons or octaves, as set theory and interval analysis are concerned with the relative relationships between distinct pitch classes, rather than intervals formed by identical tones or octave equivalents.

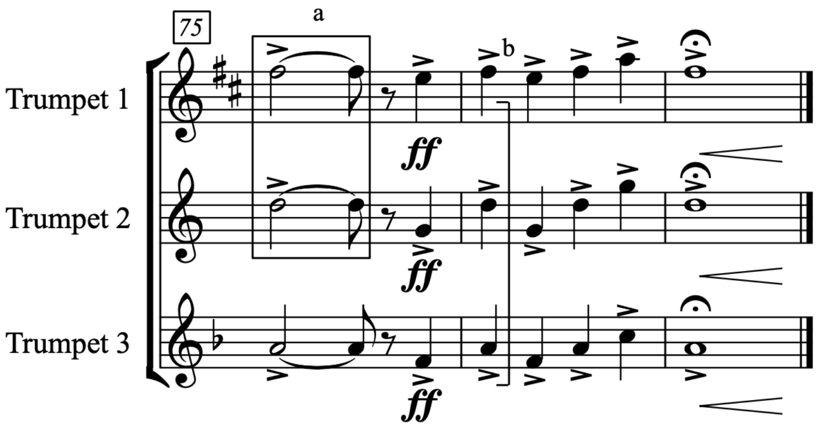
The fundamental motivation for extending unordered interval classes in atonal music stems from the analytical need to account for sonority equivalence in intervallic relationships.[5] However, this abstract simplification strategy is not without cost: in specific contexts—especially in multi-voice counterpoint or compositions with clearly directional structural layout—excessive reliance on unordered intervals may obscure interval structures that inherently possess directional meaning. In polytonal counterpoint, all such intervallic instances should be understood as *directed*—that is, as ordered intervals from set A to set B, rather than the unordered intervals emphasised in atonal music. The justification for this treatment is clear: polytonality stresses the independence and distinctiveness of pitch material between parts. It does not presume that any given interval is structurally interchangeable or invertible.

For example, in Figure 1, the interval *a*—a major third (OIC4, D–F♯) between the second and first trumpet is unambiguous in directionality and scalar attribution, as both parts lie in the same register and the span is smaller than a tritone. In contrast, the interval *b*—a minor sixth (OIC8, A–F♯) between the third and first trumpet is different: it exceeds the span of a tritone. Interpreting it, under the concept of unordered interval classes, as an inverted major third (OIC4, F♯–A) would wrongly assign F♯ to the diatonic

5 For a detailed discussion on interval relations in modern music, see (Hanson, 1960).

scale set of the third trumpet. Such an interpretation would conflict with the sustained contrapuntal structure (F major–D major), thereby undermining an accurate judgment of its tonal stability.

Figure 1 - Benjamin Britten, *Fanfare for St. Edmundsbury*, 1959



When examining polytonal harmonic intervals formed between two diatonic scale sets, and since melodic intervals are not involved, each pitch interval must be formed by selecting one element from set A and another one from set B; in other words, combinations between pitches within the same set are not considered. Under this setup, the number of possible interval pairs (a, b) formed by any two diatonic scale pairings is always the same. The Cartesian product *S* represents all possible intervallic instances between the two sets. Assuming that each scale set contains seven pitches, the total number of intervallic instances is 49.

The Interval Content in Polydiatonic Counterpoint

In set theory, the complete set of (unordered) interval-class representatives corresponding to the absolute value differences between all pairs of elements within a given pitch-class set is referred to as the set's interval-class content.[6] A six-digit integer typically represents this content, known as the *absolute pitch interval-class* (APIC) vector (Schuijer, 2008, pp. 47-48). A more general name for this concept is the interval vector (Forte, 1973,

6 David Lewin termed this concept the interval function, see (Lewin, 1959).

p.209), or, more precisely, the interval-class vector (Straus, 2016, p. 21). For example, the diatonic set 7-35 has the APIC vector [254361], in which each value (from the first digit 2 to the last digit 1) corresponds to the number of occurrences of the unordered interval classes IC1 through IC6 within the set, reflecting the set's potential to form each interval class.

In an atonal context, the APIC vector—based on the concept of unordered interval class—is primarily used to describe the intervallic distribution among element pairs within a specific pitch-class set. However, this concept often proves difficult to apply directly in the context of polytonal music. When analysing relationships between two or more sets, set theory places greater emphasis on their musical coherence (Schuijer, 2008, p. 4). In general, such coherence may be assessed through dimensions such as difference, similarity, or complementarity. Nevertheless, the APIC vector itself does not directly reveal the intervallic relationships between element pairs across multiple sets, which limits its applicability when addressing inter-set interactions.

In polytonal structures, the diatonic scalar sets associated with different tonal levels function as relatively independent entities within their respective voices, while also interacting with one another on the harmonic level. The construction of such harmonic intervals essentially involves ordered pairings between elements from two distinct sets, A and B, rather than the free combinations of internal elements formed by the simple union of sets A and B.

This distinction can be illustrated with a simple example. Let sets A = {C, E, G} and B = {C, D, G}. Their union is set C = A ∪ B = {C, D, E, G}. If we consider all ordered pairings between elements of A and B, we obtain 3 × 3 = 9 pairings (e.g., C [from A] – D [from B], E – G, G – C, including identity pairings such as C – C and G – G). This pairing preserves the directional distinction between the sets (it allows for C–D but not D–C). In contrast, if we examine all ordered pairings within the union set C — that is, the Cartesian product C × C, — we obtain 4 × 4 = 16 ordered pairs (e.g., C–C, C–D, D–C, E–G, etc.). These represent all possible ordered relationships among the elements of the union, encompassing pairings both within and across the original sets, but without retaining any distinction between their sources.

Thus, the ordered pairings between two sets—A × B—differ markedly, in both quantity and structural properties, from the ordered pairings within their union—C × C; the two must not be conflated in the analysis of polytonal or contrapuntal music.[7] Adhering to this principle, the pitch pairings produced through counterpoint between different diatonic sets in polydiatonicism should be regarded as ordered intervals, which are strictly directional; their contrapuntal intervallic content therefore comprises the twelve ordered interval classes (OIC0–OIC11). Consequently, the traditional six-digit APIC vector must be expanded into a twelve-digit integer, called the *directed pitch interval class* (DPIC) vector, to represent the richer intervallic relationships characteristic of such structures in this study.

For example, in a polydiatonic counterpoint with tonic centres a fourth apart (C major–F major), the DPIC vector will be [634527254362]. Each value in this vector (from the first digit 6 to the last digit 2) corresponds to the number of occurrences of each ordered interval class (IC0 to IC11) between the two diatonic sets, reflecting the potential of this contrapuntal structure to produce various interval instances. In what follows, I will use this particular instance of polydiatonic structure (C major–F major) as a case study to examine the distributional characteristics of intervallic content in such contexts.

Results

Contrapuntal Polydiatonic Structure: C–F

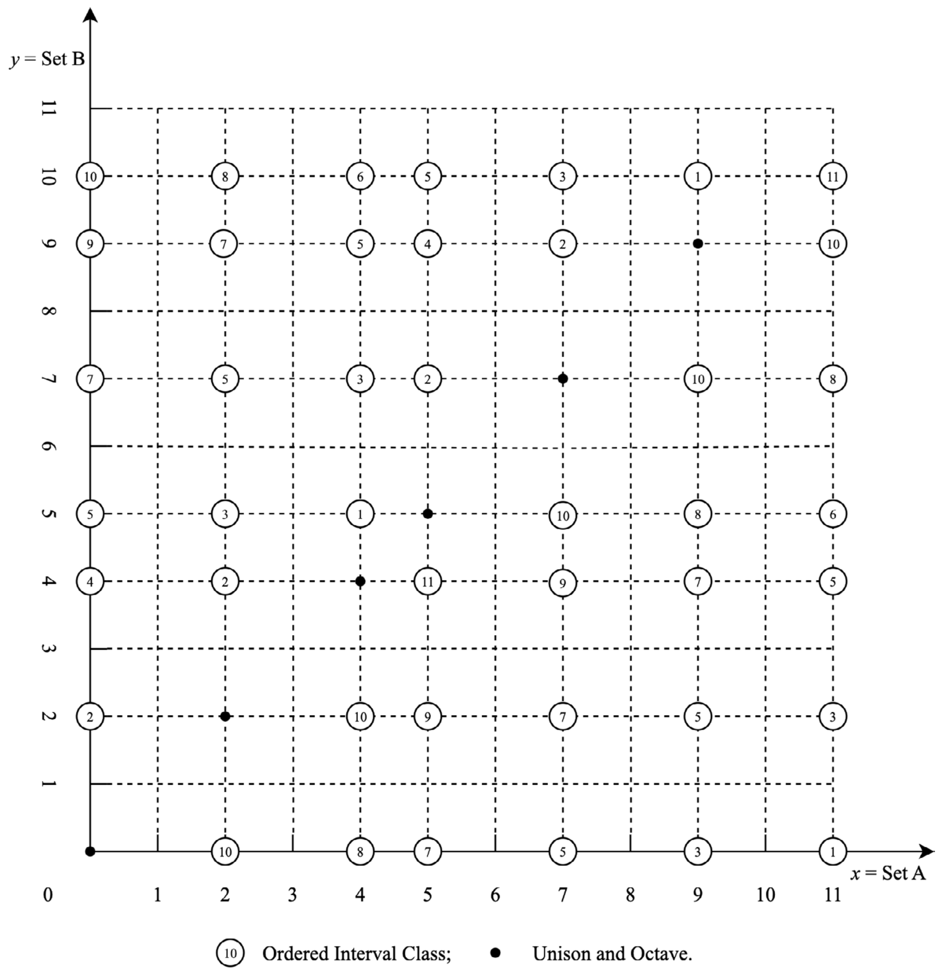
Using the C major scale as the fixed pitch-class set A, and taking each of the twelve diatonic scales in turn as the contrapuntal pitch-class set B, one can construct twelve distinct structures of contrapuntal polydiatonicism. This section selects

7 In the analyses of the "Petrushka chord" by Richard Taruskin (1996), Pieter van den Toorn (1983), and Arthur Berger (1963), for example, scale superpositions arising from contrapuntal relationships are sometimes treated as equivalent to octatonic constructions, carrying a risk of conceptual confusion. By contrast, Dmitri Tymoczko's multidimensional approach to musical structure more effectively reveals the internal logic of its polyphonic layers and pitch organization, see (Tymoczko, 2002).

the fourth-related contrapuntal structure (with the basic form C–F) as a case study to illustrate the analytical results.

In such a structure, the C major diatonic pitch-class set A = {0, 2, 4, 5, 7, 9, 11} serves as the lower fixed set, while the F major diatonic set B = {0, 2, 4, 5, 7, 9, 10} functions as the upper contrapuntal set. According to the method for calculating ordered intervals, an ordered interval pair (a, b) is formed by selecting one pitch from set A and one from set B. All possible pairings may be visualised using a coordinate-axis representation, as shown in Figure 2.

Figure 2 - Ordered interval-class matrix of the fourth-related (C–F) contrapuntal polydiatonicism



X-axis: C major scale; y-axis: F major scale; as ordered intervals, the horizontal and vertical axes are not interchangeable

As shown in the matrix in Figure 2, the contrapuntal polydiatonicism based on a perfect fourth relationship yields only a limited number of intervallic instances—far fewer than the 144 possible pairs covered in the twelve-tone system. As previously noted, in a contrapuntal relationship between two diatonic scales with fixed voice positions, the total number of interval instances that can occur is 49. Therefore, for polydiatonic music to generate a broader range of contrapuntal intervals, it must resort to tonal modulation, superimposition of tonal structures, or the complete abandonment of the diatonic system. However, regardless of the strategy employed, from the perspective of intervallic content alone, the inherent limitations in pitch material mean that this form of polydiatonic counterpoint can only endlessly "approach the domain of atonality," without ever achieving the fully saturated chromatic ideal.[8]

Next, by statistically organising the various interval classes within this contrapuntal structure, we arrive at its DPIC vector, as shown in Table 1.

Table 1 – DPIC vector of contrapuntal polydiatonicism based on a perfect fourth relationship (C–F)

Interval Class	Interval Content	DPIC Vector
IC0	(0,0), (2,2), (4,4), (5,5), (7,7), (9,9)	6
IC1	(4,5), (9,10), (11,0)	3
IC2	(0,2), (2,4), (5,7), (7,9)	4
IC3	(2,5), (4,7), (7,10), (9,0), (11,2)	5
IC4	(0,4), (5,9)	2-
IC5	(0,5), (2,7), (4,9), (5,10), (7,0), (9,2), (11,4)	7+
IC6	(4,10), (11,5)	2-
IC7	(0,7), (2,9), (5,0), (7,2), (9,4)	5
IC8	(2,10), (4,0), (9,5), (11,7)	4
IC9	(0,9), (5,2), (7,4)	3
IC10	(0,10), (2,0), (4,2), (7,5), (9,7), (11,9)	6
IC11	(5,4), (11,10)	2-

"7+" indicates that this interval class contains the highest number of interval instances (7 types).
"2-" indicates that this interval class contains the fewest types of interval instances (2 types).

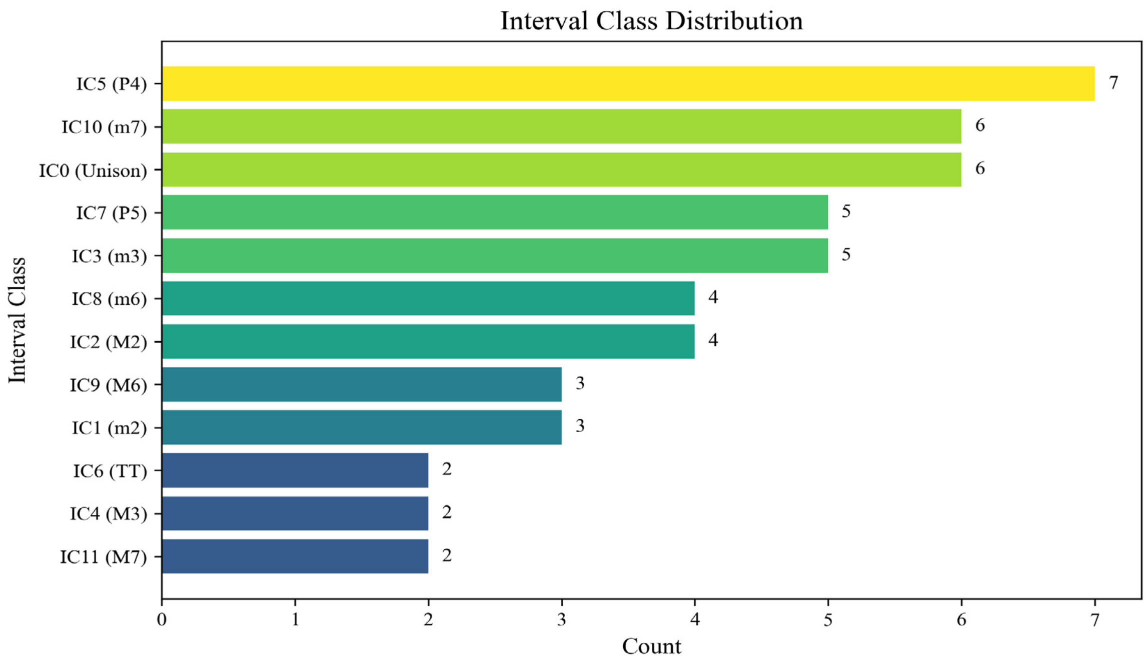
8 This implies a flaw in the view of Darius Milhaud (1923): no amount of key superposition can generate an interval inventory equivalent to that of twelve-tone music with the same number of voices.

Table 1 provides two types of information regarding this contrapuntal structure: first, the interval content, which encompasses all possible intervallic instances across the interval classes; and second, the DPIC vector, which indicates the number of occurrences for each specific interval class. Beyond its statistical value, the DPIC vector serves as a highly practical tool in compositional contexts. Certain diatonic pitch-class sets may exhibit greater advantages over others in terms of specific intervallic properties—such as the relative abundance of consonant or dissonant intervals—and these tendencies are often reflected in their interval class vectors (Huron, 1994). Composers, guided by varying aesthetic or structural considerations, may thus be influenced by such intervallic characteristics in their deliberate selection of pitch materials.

It is not difficult to understand that Huron's investigation into set characteristics begins precisely with the differences in how various sets manifest in their interval class vectors. Similarly, in contrapuntal polydiatonicism, the DPIC vectors generated by different contrapuntal structures exhibit significant divergence. These differences are evident not only between distinct diatonic scalar sets but also in the distribution across individual interval classes. As shown in Table 1, every interval class in this configuration contains at least two interval instances, suggesting that even within the limits of diatonic pitch materials, a relatively rich network of intervallic relationships can emerge. Among these, IC5 (perfect fourth) appears most frequently, with a total of seven instances (marked with a "+" in the upper-right corner of the table), indicating its primary role in this contrapuntal configuration. In contrast, IC4 (major third), IC6 (tritone), and IC11 (major seventh) occur only twice each—the fewest among all interval classes (marked with a "–"). The remaining interval classes show a degree of symmetrical pairing: IC0 (unison) with IC10 (minor seventh); IC3 (minor third) with IC7 (perfect fifth); IC2 (major second) with IC8 (minor sixth); and IC1 (minor second) with IC9 (major sixth). The underlying cause of these pairings appears to relate to a certain symmetry between the tonic centres of the two diatonic collections—though due to space limitations, this point will not be explored further here.

It can be observed that the DPIC vector produced by contrapuntal polydiatonicism shares certain similarities with the APIC vector characteristics of individual pitch-class sets in Forte's set theory. For instance, within the overall structure, the vectors reveal both quantitative differences among interval classes and, at times, partial similarities in the frequency of certain interval classes. This pattern of interval distribution may influence a composer's decisions in selecting pitch-class sets (or tonalities) and determining contrapuntal relationships, thereby allowing for a more effective coordination between structural logic and perceptual experience during the compositional process. The distinctive intervallic distribution of this contrapuntal polydiatonic structure can be illustrated by ranking the interval classes according to their instance count, as shown in Figure 3:

Figure 3 – Interval-class distribution in polydiatonic counterpoint (C-F)



The distribution of interval instances reveals that the interval classes can be roughly divided into six levels:

- Level 1: IC5 (perfect fourth), with seven instances—the highest count—indicating its central role.
- Level 2: IC0 (unison) and IC10 (minor seventh), each with six instances, both abundantly present.
- Level 3: IC3 (minor third) and IC7 (perfect fifth), each with five instances, are relatively frequent.
- Level 4: IC2 (major second) and IC8 (minor sixth), each with four instances, moderate in number.
- Level 5: IC1 (minor second) and IC9 (major sixth), each with three instances, are less frequent.
- Level 6: IC4 (major third), IC6 (tritone), and IC11 (major seventh), each with only two instances—indicating their marginal status.

It is not difficult to observe that the first three levels together account for 29 interval instances—more than half (approximately 59.18%) of the total 49. This indicates that, within this polydiatonic contrapuntal structure. However, the first three levels include only five interval classes—fewer than the seven found in the remaining levels—their substantial share of total interval instances demonstrates a higher degree of structural involvement and combinatorial density. Specifically, IC5, IC0, IC10, IC3, and IC7 may be regarded as *prominent interval classes*, exhibiting greater combinatorial activity and structural significance within this contrapuntal setting. By contrast, IC2, IC8, IC1, IC9, IC4, IC6, and IC11 occur less frequently and may thus be seen as *non-prominent*, occupying a more peripheral role in the structure. This distribution reflects a functional differentiation and hierarchical weighting among interval classes in contrapuntal polydiatonicism, suggesting that certain intervallic relationships play a more essential role in shaping the structural integrity of the system.

As the interval class with the highest number of instances among the prominent interval classes, IC5—classified in the first level—holds significance not only due to its high frequency of occurrence, but also because it reflects the underlying structural function of the perfect fourth relationship within this polydiatonic contrapuntal setting. In other words, IC5 reveals a deep structural association with the perfect fourth, indicating its foundational role in the construction of the contrapuntal framework. Its status should therefore be understood as *primary*, or as the *prime interval class*, occupying a central position within the overall intervallic distribution. In contrast, the sixth level's IC4, IC6, and IC11—categorized as non-prominent interval classes due to their minimal frequency—should be regarded as *subordinate*, or *secondary interval classes*, within this structure.

Of course, this merely observes the inherent combinatorial logic embedded within these pitch collections from a naturalistic perspective, demonstrating that interval classes in counterpoint exhibit a hierarchical structure in theoretical terms. This does not imply that real composition must conform to the limitations of such combinatorial potential. On the contrary, composers often make deliberate choices about intervallic content based on varying theoretical frameworks and aesthetic goals, thereby surpassing these latent structural constraints. This is entirely understandable and acceptable. Human emotional expression and aesthetic decision-making are inherently complex processes that are seldom wholly governed by the natural combinatorial possibilities of pitch material. In this sense, the relationship between natural structure and human creativity should be understood as a dynamic interaction: the former offers a set of probabilistic tendencies, while the latter—through selection, transformation, and reconstruction—endows the work with creative meaning and expressive power.

Some Characteristics of the DPIC vector in Contrapuntal Polydiatonicism

A Brief Comparison with the APIC vector

In the previous discussion, the basic form of a perfect fourth relationship (C–F) in contrapuntal polydiatonicism was used as an example to analyse localised characteristics in terms of interval classes and DPIC vectors. Suppose the same analytical method is extended to the remaining eleven basic structures. In that case, one can derive the complete set of DPIC vectors for all twelve forms of contrapuntal polydiatonicism (see Table 2). It is evident that unified tonal counterpoint (C–C), as a long-standing musical tradition, is also included within the scope of contrapuntal polydiatonicism. In this context, it is regarded as a special case or the initial structure of contrapuntal polydiatonicism, sharing the same structural features as the other basic polydiatonic configurations. On this basis, it becomes possible to observe and grasp the overall characteristics of contrapuntal polydiatonicism at a higher-level perspective rooted in historical continuity.

Table 1 - DPIC vectors of twelve basic forms of polydiatonic counterpoint

Tonal Relationship	Basic Polydiatonic Form	DPIC Vector
11	C-B	254362634527
10	C-B♭	543626345272
9	C-A	436263452725
8	C-A♭	362634527254
7	C-G	626345272543
6	C-F♯	263452725436
5	C-F	634527254362
4	C-E	345272543626
3	C-E♭	452725436263
2	C-D	527254362634
1	C-D♭	272543626345
0	C-C	725436263452

It is not difficult to observe that the DPIC vectors of the twelve basic forms of contrapuntal polydiatonic structures all differ from one another. Despite this diversity in value distribution, several unifying features still emerge among them. First, in all structures, the *primary interval class*—that is, the interval class with the most significant number of interval instances—has a fixed vector value of 7 and always appears as a single item. This interval class typically reflects the contrapuntal relationship between the tonics of the two diatonic scales involved. Second, each structure consistently contains exactly three *secondary interval classes*; the interval class with the lowest number of interval instances—has a fixed vector value of 2 and always appears as three items. The remaining eight interval classes are distributed in pairs with interval counts of 3, 4, 5, or 6, forming a relatively balanced pattern of arrangement.

When compared with the APIC vectors of single sets in set theory, this result further highlights the uniqueness of contrapuntal polydiatonic structures. In set theory, the vector values of the six unordered interval classes exhibit a much wider range of variation, with maximum values reaching six and minimum values potentially being 0, indicating the possibility of certain interval classes being entirely absent. At the same time, the interval-class vectors of individual sets possess much greater flexibility and complexity in their structural composition: the number of instances in each interval class may be either identical or completely different.

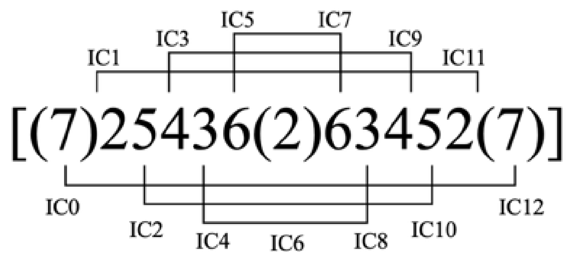
By contrast, the DPIC vectors of contrapuntal polydiatonic structures exhibit greater stability and coverage. The maximum vector value consistently reaches 7, while the minimum never falls below 2, meaning that no interval class is ever absent in any given structure. To a certain extent, this feature may be regarded as an advantage—it offers composers a more balanced and reliable intervallic foundation, avoiding the structural incompleteness found in some other systems. However, it is important to note that this structural balance also limits the potential for dynamic contrast between different contrapuntal configurations. For instance, even in unified tonal counterpoint, IC6 (the tritone) appears only twice—already the minimum. In contrast, in tritone-based structures, IC6 seems to be up to 7 times higher, yet the difference between them is still only 5. The differences among other contrapuntal structures are even less pronounced, revealing a specific constraint in the expressive tension across interval classes within diatonic-scale-based contrapuntal structures.

The Cyclic and Symmetrical Structures of DPIC vectors

Moreover, the DPIC vectors of these twelve basic forms display a pronounced cyclic and symmetrical ordering. Cyclicality here means the following: taking the unisonal pairing (C–C) as the starting point, whose DPIC vector is [725436263452], if the tonic of the contrapuntal voice (set B) is transposed upward by *n* semitones in twelve-tone equal temperament, the entire DPIC vector likewise shifts *n* places to the right. For example, transposing by one semitone (producing a minorsecond relationship) yields the DPIC vector [272543626345] —that is, each element of the original sequence moves one step to the right, and the initial digit cycles to the end, forming a stable numerical loop.

Corresponding to this cyclic behaviour, the vectors also exhibit apparent symmetry. Using the unified tonal structure (C–C) once more, its DPIC vector reveals a mirror structure about IC6 (and, equivalently, about IC0) near the center: the first half and the second half display numerically and directionally symmetric distributions. This symmetry presupposes the complete hierarchy of interval classes and becomes fully apparent only when the commonly omitted IC12 (the octave) is reinstated (see Figure 5).

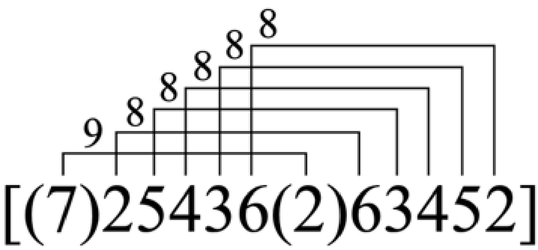
Figure 5 - The symmetrical structure of contrapuntal polydiatonic DPIC vector (C-C)



Apart from IC6 (or IC0), which lies at the centre of symmetry and thus cannot be paired with any other interval class, all other interval classes can be grouped into complementary pairs: IC0 and IC12, IC1 and IC11, IC2 and IC10, IC3 and IC9, IC4 and IC8, IC5 and IC7. Naturally, this characteristic shifts according to the contrapuntal relationship that defines each structure's cyclic ordering. For instance, in the case of the minor second relationship, the axis of symmetry in the natural-pitch polytonal counterpoint shifts one position forward relative to the unisonal case, becoming IC7 (or IC1).

It becomes evident that the axis of symmetry holds a particularly significant position within the DPIC vector. It is often either the primary interval class of the corresponding polydiatonic counterpoint or one of the three secondary interval classes. Around these two axes of symmetry, we also observe a kind of complementary relationship between specific pairs of interval-classes and their values—though this relationship is not always perfectly consistent (see Figure 6).

Figure 6 - Complementary relationship between pairs of interval-class and its values in DPIC vector (C-C)



It can be observed that the two axes of symmetry—IC0 and IC6—differ by six interval classes, and when paired, their corresponding values sum to 9 (e.g., 7 + 2 for IC0 and IC6). In contrast, other interval class pairs that are also 6 classes apart (e.g., IC1–IC7, IC2–IC8, etc.) consistently yield a value sum of 8 (e.g., 2 + 6 for IC1 and IC7, 5 + 3 for IC2 and IC8). This reveals a stable distribution pattern: across all basic forms, one of the three secondary interval classes is always bound to the primary interval class, forming a pair whose values sum to 9. This interval class may be referred to as the *secondary interval class associated with the primary interval class*. The other two secondary interval classes each pair with an interval class with a vector value of 6, forming regular combinations that sum to 8. This pattern is consistently present across all twelve contrapuntal polydiatonic structures, allowing for a finer distinction among the functional roles of the three secondary interval classes.

This characteristic also provides a theoretical basis for a simplified representation of the DPIC vector. If the first six values are known, and the position of either the primary interval class or its associated secondary interval class is identified, then—based on the above rule—the value of the corresponding paired interval class (separated by 6) can be calculated by subtracting the known value from either 9 or 8, thereby reconstructing the complete twelve-digit integer. In Table 2, the full twelve-digit DPIC vectors are presented without omission for clarity. However, the first six values of each are underlined to highlight the segment containing the primary interval class or its associated secondary component.

A Case Study from Darius Milhaud's The Fourth Chamber Symphony

In the third movement of Darius Milhaud's *String Symphony No. 4*, a rich and inventive exploration of polytonality unfolds, grounded in Baroque style and fugal writing. This movement serves as a notable example of employing polydiatonic counterpoint as a core compositional technique (Kelly, 2017, p. 171). As an analytical case, I have selected measures 28 to 37 of this movement (see Figure 7), focusing on the melodies of the first and second violin parts. The analysis centres on polytonal counterpoint based on diatonic scales, intending to illustrate how polydiatonic counterpoint assumes a leading role in composition through the construction of contrapuntal intervals.

Figure 7 - Darius Milhaud's *The Fourth Chamber Symphony, Movement III,* mm.28-37

Table 3 - Contrapuntal intervals and DPIC vector statistics

Interval Class	Interval Name	Interval Instances	DPIC Vector	
			IV^1	IV^2
0	Unison/Octave	(0, 0, 1); (8, 8, 1); (4, 4, 2); (5, 5, 1); (2, 2, 1)	5	6
1	Minor 2nd	(4, 5, 1); (11, 0, 2); (9, 10, 1)	3	3
2	Major 2nd	(0, 2, 5); (2, 4, 4); (5, 7, 1); (7, 9, 2)	4	4
3	Minor 3rd	(9, 0, 1); (4, 7, 1); (2, 5, 1); (11, 2, 1)	4	5
4	Major 3rd	(0, 4, 2); (5, 7, 1)	2	2
5	Perfect 4th	(0, 5, 3); (9, 2, 2); (2, 7, 1); (5, 10, 1); (11, 4, 2)	5	7^+
6	Tritone	(4, 10, 1)	1	2
7	Perfect 5th	(0, 7, 2); (7, 2, 2); (5, 0, 1); (9, 4, 1)	4	5

8	Minor 6th	(11, 7, 3); (4, 0, 2); (9, 5, 2)	3	4
9	Major 6th	(0, 9, 2); (7, 4, 2); (5, 2, 2)	3	3
10	Minor 7th	(0, 10, 3); (7, 5, 2); (9, 7, 2); (2, 0, 2); (4, 2, 2)	5	6
11	Major 7th	(5, 4, 2); (11, 10, 1)	2	2-

IV[1]: The DPIC vector distribution that was adopted in the excerpt.
IV[2]: The theoretical DPIC vector distribution of this polydiatonic counterpoint structure.

Excerpted from Milhaud's Chamber Symphony No. 4, movement III. The second violin part (in C) and the first violin part (in F) form a perfect fourth relationship. Each interval instance is represented as a triplet (a, b, c), where a is a pitch from the lower part's diatonic scale set (C), b is a pitch from the upper diatonic scale set (F), and c indicates the number of times this instance appears in the passage.

Table 5 presents a statistical summary of all interval classes, their corresponding instances, and the actual DPIC vector distribution found within this passage. It is worth noting that intervals formed through *heterorhythmic* or *contrarythmic* counterpoint are counted using the smallest rhythmic unit. In contrast, portions where one voice rests and no interval can be formed are excluded from the tally.[9] In the actual distribution of intervals, the most frequently occurring interval classes (IC0, IC5, IC10, IC7, IC3) all fall under *prominent interval classes*; by contrast, those with lower frequency (IC6, IC11, IC1, IC9, IC8) are consistently *non-prominent interval classes*. This outcome closely aligns with the DPIC vector characteristics of the polydiatonic structure based on the perfect fourth relationship. It thus becomes evident that the structure of polydiatonicism imposes an apparent normative influence on musical realisation, essentially determining the range of usable intervallic material. Neglecting the contrapuntal properties inherent to the structure will inevitably pose limitations and challenges in actual composition.

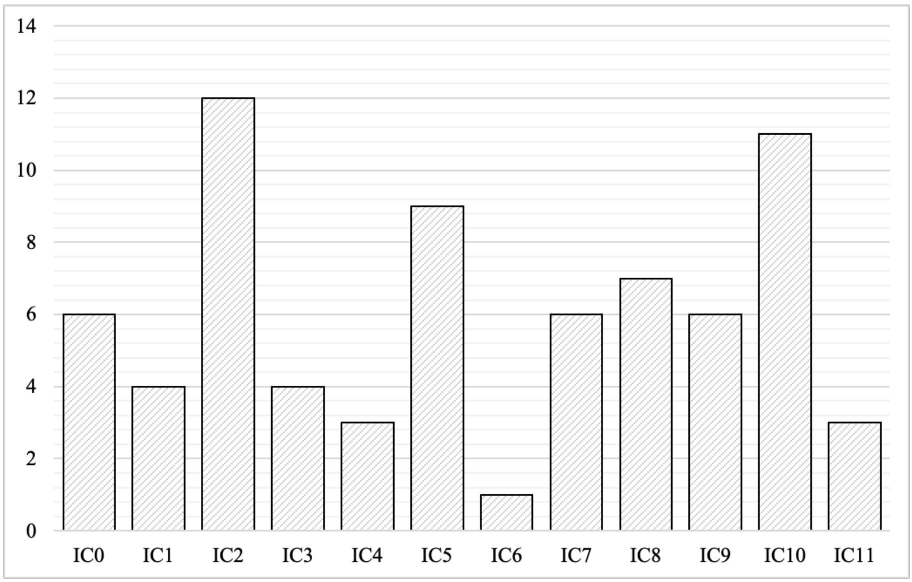
9 For the concepts of *heterorhythmic* and *contrarythmic* counterpoint, see (Berry, 1987, pp. 191–195).

The actual interval content found in a musical work—namely, its realised DPIC vector values—offers valuable insight into the functional status of a given interval class within the music. For example, even when each contains five interval instances, IC0, IC5, and IC10 exhibit varying degrees of alignment with their theoretical expectations. Specifically, IC5, which theoretically includes seven instances and functions as the primary interval class, does not fully demonstrate its structural advantage in this passage. Although IC0 and IC10 likewise fall short of presenting their complete interval profiles, their omission ratios (1/5) are slightly lower than IC5's (2/7). In other words, despite the identical number of actual instances, IC5 appears to face comparatively greater limitations in practical use.

Moreover, interval classes with the same or similar theoretical counts may behave differently in actual musical contexts. For instance, although the secondary interval class IC11 has two instances and IC6 only one, the former is clearly employed with greater frequency and significance in this passage.

Another, more intuitive method of comparison is to examine the total number of instances for each interval class in total, see Figure 8.

Figure 7 - Distribution harmonic intervals: Darius Milhaud, *The Fourth Chamber Symphony, Movement III*, mm.28-37



From the perspective of interval class frequency, the composer's choice of a perfect fourth relationship as the basis for polydiatonic counterpoint seems to reflect a preference for IC2 and IC10—IC5 and IC8 may also be considered in this context. These interval classes appear with notably higher frequency than others. By contrast, IC6, IC4, and IC11 occur the least. This distribution pattern, to a certain extent, reflects the structural features of diatonic-scale polytonality based on the perfect fourth relationship: for instance, the frequent use of IC10 and IC5 aligns with their theoretical status as prominent interval classes, while the infrequent appearance of IC4, IC6, and IC11 is consistent with their classification as non-prominent interval classes.

However, IC2 and IC8—especially IC2—stand out as exceptions: although they each contain only four instances in the theoretical context and are considered non-prominent interval classes, IC2 emerges as the most frequently used interval in this passage, clearly diverging from its expected structural behavior. Another notable exception is IC3: although classified as a prominent interval class, it appears less frequently—and with fewer distinct instances—than several non-prominent interval classes such as IC1, IC9, IC2, and IC8.

This suggests that, in composing this polytonal passage, the composer deliberately limited the usage of IC3, despite its structural advantage, while significantly expanding the use of IC2. This choice does not come without cost: the "enhancement" of non-prominent interval classes could only be achieved through the repeated use of a few specific interval instances. In the case of IC2, such repetition is reflected in the frequent foreground recurrence of the (0, 2) and (2, 4) as interval instances.

Discussion and Conclusion

Any contrapuntal polytonal combination constructed from diatonic scales will, by its contrapuntal logic, constrain a specific set of possible harmonic intervals. These diatonic scales, each with its tonal orientation, can be understood as a kind of compositional "palette" preselected by the composer—a limited set of pitch resources, where how pitches are combined may be likened to a painter mixing pigments. The available colours on the palette determine the accessibility of certain hues or shades; likewise, the pitch-class collections derived from diatonic scales determine the relative frequency of particular interval classes. This analogy can be extended using the concept of the APIC vector from set theory: the DPIC vector resulting from contrapuntal polydiatonicism reveals the distribution of interval classes within the given structure, thereby offering a helpful basis for understanding its harmonic potential and for assisting composers in selecting appropriate pitch materials.

Moreover, the system developed from two-voice contrapuntal theory is not confined to the context of "bitonal" counterpoint; it also possesses the potential to be extended to more complex structures, such as polytonal compositions involving three or more voices. A three-voice texture can be understood as a combination of three two-voice contrapuntal relationships. At the same time, a four-voice counterpoint may be viewed as the superposition of six such pairs. This approach may thus be regarded as a continuation and expansion of the traditional framework of multi-voice contrapuntal theory.

In conclusion, the distribution of harmonic intervallic contents in contrapuntal polydiatonicism exerts a fundamental influence on compositional practice. The variety of intervallic types generated by the combination of diatonic scales based on different tonal centres essentially reveals this principle: prominent interval classes tend to prevail in the vertical presentation of contrapuntal intervals. This phenomenon reflects the comprehensive regulatory power of tonal relationships over contrapuntal outcomes and suggests that an inherent theoretical rationale supports the choice of tonal centres between contrapuntal voices. Although in actual composition individual non-prominent interval classes may sometimes be given higher priority than prominent ones, this does not imply the failure of the underlying theory, but instead points to the composer's deliberate control over the intervallic inventory (Huron, 1991, 1994). In fact, such control not only avoids excessive complexity but also offers analysts valuable clues that bring them closer to the composer's creative intent.

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Acknowledgement

We would like to express our heartfelt gratitude to the two anonymous reviewers for their many valuable comments and suggestions, which greatly helped improve the quality of this study.

Authorship contribution

Conceptualization, X.C.; methodology, X.C.; formal analysis, X.C.; investigation, X.C.; resources, X.C.; writing—original draft preparation, X.C.; writing—review and editing, M.S.; supervision, M.S. All authors have read and agreed to the published version of the manuscript.

Publisher

Federal University of Goiás. School of Music and Performing Arts. Graduate Program in Music. Publication in the Portal of Periodicals UFG.

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